DIAZ-FITZGERALD TIME DOMAIN (D-FTD) TECHNIQUE APPLIED TO ELECTROMAGNETIC PROBLEMS

Franco De Flaviis, Massimo Noro* and Nicolaos G. Alexopoulos

University of California at Los Angeles, Department of Electrical Engineering
*Department of Chemistry and Biochemistry, Los Angeles, CA 90024

ABSTRACT

A new approach for the solution of electromagnetic problems is presented and tested. The method is based of the discretization of Maxwell's equations, but it differs from the FDTD implementation in that it exploits a different scheme to model wave propagation. Vector and Scalar potential are used instead of the magnetic field, to propagate the electric field. This implementation provides a *condensed node* representation for the electric field, offers a natural way to treat interfaces, and allows to model Debye relaxation media, *avoiding convolution*. Additionally a mechanical analog can be devised.

1. INTRODUCTION

The Diaz-Fitzgerald Time Domain (D-FTD) technique is based on the discretization of Maxwell's Equations via the introduction of scalar and vector potential. If space is discretized, the time evolution of electric and magnetic field in each mesh position can be correctly described by a leap-frogging scheme between the electric field and the vector potential. The formulation presented here differs substantially from the one presented by Leubbers[1], since in this case the electric field is accessible at each step of iteration, so it can also be used to satisfy appropriate boundary conditions. For the general 3-D case the coupled equations between Electric Field, Vector Potential and Scalar Potential[2] are :

$$\begin{cases} \dfrac{\partial \phi}{\partial t} = -\dfrac{1}{\varepsilon\mu}\nabla \cdot \mathbf{A} \\ \dfrac{\partial \mathbf{E}}{\partial t} = \dfrac{1}{\varepsilon\mu}\left[\nabla(\nabla \mathbf{A}) - \nabla^2 \mathbf{A}\right] \\ \dfrac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} - \nabla\phi \end{cases} \tag{1}$$

A suitable discretization scheme must be chosen in order to obtain a stable approximation. Typically in EM problems, the central difference scheme leads to stable discretizations: this approach is pursued. Excitation of the system is done as in other time domain methods: an electric field pulse is set, and within a single run of the code one can obtain information over a wide spectrum of frequencies. The data obtained from the pulsed method can be processed using a Fast Fourier Transform (FFT) or a Discrete Fourier Transform (DFT).

2. TWO DIMENSIONAL FORMULATION FOR TM PROBLEMS

Let's consider an EM problem where only E_Z, H_X and H_Y exist (TM_Z). The set of Equations (1) can be then reduced to:

$$\begin{cases} \dfrac{\partial E_z}{\partial t} = -\dfrac{1}{\varepsilon\mu}\left[\dfrac{\partial^2 A_z}{\partial x^2} - \dfrac{\partial^2 A_z}{\partial y^2}\right] \\ E_z = -\dfrac{\partial A_z}{\partial t} \end{cases} \tag{2}$$

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Because of the absence of the z-dimension, all the derivatives with respect to z do not exist. Furthermore from the iterative scheme it is clear that, since $E_x=E_y=0$, necessarily $A_x=A_y=0$ and f=0. Only A_z is different from zero. As shown in [3], the Hamiltonian of this system corresponds to that of an array of spheres with fixed axis of rotation connected by rubber spheres. A similar mechanical model was first proposed by Fitzgerald[4] in 1835 and recently implemented as a numerical technique by R.Diaz[5]. Notice that it is not necessary at this point to introduce the mechanical analogy, but we believe that it provides a clearer understanding of the details of the propagation of the field, and it gives further insight of the quantities involved in the simulation, especially when complex phenomena as multiple Debye relaxations are present.

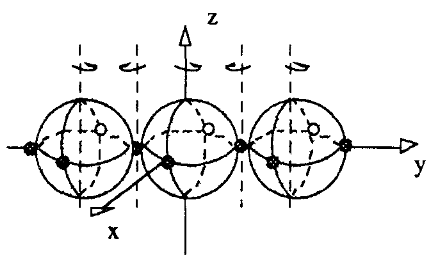

Fig. 1 Array of rigid spheres connected by rubber spheres

Consider an ordered array (for clarity only one line is shown) of spheres connected to each other with small rubber spheres as described in Figure 1. Each sphere is connected to the four neighbor spheres by rubber spheres which are assumed not to slip. The rotational motion of the pulley in position (i,j) is therefore coupled to the motion of the spheres (i-1,j), (i+1,j), (i,j-1) and (i,j+1). In fact the sphere (i,j) with moment of inertia I(i,j) is free to rotate around the z-axis with angular velocity $\omega = \partial\theta(i,j)/\partial t$, where $\theta(i,j)$ is the rotation angle with respect to a common reference axis. Let the radius of the sphere be a and the distance from two adjacent spheres be $2a$, then a difference in rotation angle between two adjacent spheres $\Delta\theta$ generates a tension $T = 2ka\Delta\theta$ in the rubber sphere connecting them. This is manifested as a tangential force (F_{1x},F_{2x},F_{1y},F_{2y}) on the rigid sphere. Combining the tensions due to the four neighbors together with Newton's second law $T = I\alpha$ where $\alpha = \partial\omega/\partial t$, and discretizing in time[5] we therefore obtain:

$$\alpha^{n+1}_{i,j} = \frac{k_{i,j}a^2}{I_{i,j}}\left\{4\theta^n_{i,j} - \theta^n_{i+1,j} - \theta^n_{i-1,j} - \theta^n_{i,j+1} - \theta^n_{i,j-1}\right\} \qquad (3)$$

$$\omega^{n+1}_{i,j} = \omega^n_{i,j} + \alpha^n_{i,j}\delta t \qquad (4)$$

$$\theta^{n+1}_{i,j} = \theta^n_{i,j} + \omega^{n+1}_{i,j}\delta t \qquad (5)$$

where the superscript n indicates the time-step. Note that in this model both the moment of inertia $I_{i,j}$ and the rubber band force constant $k_{i,j}$ depend on the particular pulley. If we identify the dielectric constant with some kind of "electric mass" of the sphere, and the magnetic permeability with some kind of "rubber elasticity", we can write

$$\begin{cases} I = \varepsilon \\ k = \mu^{-1} \\ a = (\Delta s)^{-1} \end{cases} \qquad (6)$$

so that the quantity ka^2/I becomes $1/\Delta s^2\varepsilon\mu$ and we can substitute $]E/]t,E,A$ with α,ω,θ to complete the analogy.

Since we have a handle on the electric field, all the absorbing boundary conditions devised for FDTD technique, involving E, can be implemented here too, with no additional effort. One difference between the FDTD and the D-FTD technique, is the treatment of the metal boundary. In the FDTD code the tangential component of the electric field has to be set to zero on the last two layers of the computational grid in the metal, while in the D-FTD it is sufficient to use only one layer.

3. LOSSY MATERIAL FORMULATION

In a real medium where $\varepsilon \neq \varepsilon_0$ we simply increase the moment of inertia of the pulleys, while a dielectric lossy material can be easily modeled by immersing the spheres in a viscous fluid bath, and therefore by adding a viscous damping term to Equation (4) which becomes.

$$\omega^{n+1}_{i,j} = \omega^n_{i,j} + \alpha^{n+1}_{i,j}\delta t - \gamma_\epsilon \omega^n_{i,j}\delta t \qquad (7)$$

It is easy to show that a simple relation exists between the "viscous" coefficient and the electric losses $\gamma_e = \sigma_e / \varepsilon_0 \mu_0$

Now consider a diamagnetic material with no losses: the implementation can be obtained by decreasing the rubber-band elasticity ($\mu \neq \mu_0$), while a diamagnetic lossy material can be modeled via introduction of simple friction

$$\vartheta_{i,j}^{n+1} = \vartheta_{i,j}^{n} + \omega_{i,j}^{n+1}\delta t - \gamma_m \vartheta_{i,j}^{n}\delta t \qquad (8)$$

In an analogous way it is possible to derive a simple relation between the friction coefficient and the magnetic losses $\gamma_m = \sigma_m / \mu_0$

4. APPLICATIONS

In this section we present the solution of a typical eigenvalue problems. A resonator is excited with an electrical field pulse, and, after steady state is reached, by means of DFT, the resonances frequencies are extracted. These are compared with the exact theoretical values for the m,n mode calculated from:

$$f_{m,n} = \frac{c}{2\pi}\sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \qquad (9)$$

where a and b are the linear active dimensions of the resonator and c is the speed of light. The excitation electrical field is directed along the z-axis, therefore the 2-D TM model must be used. The space is discretized using 33x33 cells; the dimension of each cell is $2.0 \cdot 10^{-4}$ m. A gaussian pulse in shape is applied in the geometrical center of the resonator. The 40 time step pulse corresponds to a width of 20 psec. This simulation is time stepped for a long enough time so that a steady state is reached, typically 8000 steps. The DFT analysis was performed using 200 points to represent a span of 80 GHz centered at 50 GHz, to locate all the excited modes. Results of this simulation is shown in Fig. 2

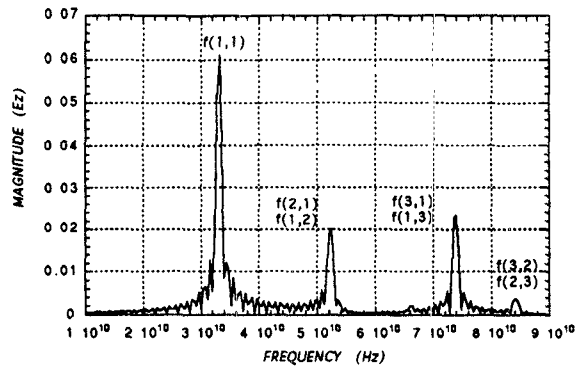

Fig. 2 DFT results for the square resonator

Successively with the same number of points we center the DFT around the fundamental mode frequency (f_{10}) using a span of 0.2 GHz, corresponding to a resolution of 10^{-3} GHz. The computer solution locates the position of the fundamental mode with an error less than 0.1%.

Particular attention must be used to determine the actual dimensions of the resonator according to the location of the x and y components of the field in the fundamental cell. In the D-FTD technique E has a condensed node representation as shown in Fig. 3,

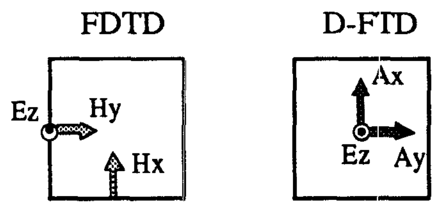

Fig. 3 (a) Yee cell field configuration
(b) D-FTD field configuration

which differs from the FDTD Yee elementary cell[6], where different components of the same field are located in different positions within the cell. The D-FTD technique offers an intuitive understanding of the actual width of the active resonator. Once the metallic boundaries are set (in the middle of the cell), the resonating cavity is measured from cell-center to cell-center. This example confirms the validity of the technique and also shows that the same accuracy as FDTD can be obtained using more natural boundary conditions.

Several other practical examples will be presented including applications of the method in practical highly dispersive materials, microstrip lines and scattering from composite objects.

5. CONCLUSIONS

A new way of solving electromagnetic problems using a time domain technique has been presented. Validation of results has been tested on a selected canonical two dimension electromagnetical problem. Introduction of mechanical analogies, as presented by R.Diaz[5], is exploited in the TM case. Thanks to the condensed node representation of the electric field the use of this new approach allows the implementation of natural boundary conditions for the treatment of metal and dielectric interfaces.

6. ACKNOWLEDGMENTS

The authors are indebted to Dr. R. Diaz for many useful discussions and criticism of this work. This work was supported in part by U.S. Army Grant DAAH 04-93-G-0228

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